Multifractal spectrum of branching random walks on free groups

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摘要：

A symmetric branching random walk (BRW) on a free group $\mathbb{F}$ is transient if and only if the mean offspring number r does not exceed R, the reciprocal of the spectral radius of the underlying random walk. In this regime, the limit set Λ_r—consisting of all ends of $\mathbb{F}$ to which the BRW's particle trajectories converge—is a proper random subset of the boundary $\partial\mathbb{F}$. Hueter and Lalley (2000) determined the Hausdorff dimension of Λ_r and proved that $\dim_{\mathrm{H}} \Lambda_r \leq \frac{1}{2} \dim_{\mathrm{H}} \partial\mathbb{F}$ with equality possible only when $r = R$.

In this talk, we further extend this study by conducting a multifractal analysis of the limit set Λ_r. We obtained the Hausdorff dimensions of the sub-fractals $\Lambda_r(\alpha) \subset \Lambda_r$ which consist of all ends of $\mathbb{F}$ approached by particle trajectories escaping at the rate $\alpha \in [0, 1]$. Notably, there exists a unique $\alpha(r) \in [0, 1]$ such that

$$\dim_{\mathrm{H}} \Lambda_r = \dim_{\mathrm{H}} \Lambda_r(\alpha(r)).$$

Moreover, an interesting phase transition occurs: $\alpha(r) > 0$ for $r < R$ while $\alpha(R) = 0$. Based on a joint work with Shuwen Lai and Longmin Wang.

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